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International Journal of Heat and Mass Transfer 48 (2005) 3381–3386
International Journal of HEAT and MASS TRANSFER
www.elsevier.com/locate/ijhmt

Theoretical study on impingement heat transfer with single-phase free-surface slot jets

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Received 10 July 2004; received in revised form 6 January 2005

Abstract

A theoretical analysis was conducted to characterize heat transfers from horizontal surfaces to normally impinging slot jets under arbitrary-heat-flux condition. The thermal and hydraulic boundary layers of laminar flow were divided into four regions of flow along heat transfer surfaces including a stagnation zone and three wall jet zones, from which general expressions of heat transfer coefficients were obtained. Furthermore, these results were compared with experimental and analytical data available in published literature. Good agreements were observed from the comparisons.

Keywords: Slot jet impingement; Heat transfer; Laminar boundary layer; Theoretical analysis

1. Introduction

Impinging jets have been widely used in industrial cooling or heating processes as a means of providing very high heat and mass transfer rates. In most cases air is employed as a working medium in jet impingement and a large amount of research on impinging air jets has been reported [1,2]. Recently, more attention has been directed to the study of liquid jet impingement as the heat transfer rates can be increased several orders of magnitude in comparison with that of gas jets. Research on liquid jets was stimulated by their possible application to the cooling of heat engines [3,4] or electronic devices [5–7], as well as in the thermal treatment of metals [8,9].

Two principal jet configurations are relevant to the application of jet impingements: circular jets and slot jets, but for which flow and heat transfer mechanics are distinctly different. A significant amount of research has been published on the heat and mass transfer with circular impinging jets [1,2,10,11]. Recently, however studies of heat and mass transfer with slot jet impingement have attracted more attention from scientists and engineers. Slot jet impingement becomes remarkable as it offers more beneficial features, such as higher cooling effectiveness, greater uniformity and more controllability. All these factors are suitable to countering the trend of continuously increasing heat flux and decreasing dimensions in compact electronics packages [12–16].

Schwarz [14] presented measurements and an analysis on the flow characteristics of two-dimensional jet

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0017-9310/$ - see front matter
doi:10.1016/j.ijheatmasstransfer.2005.02.027

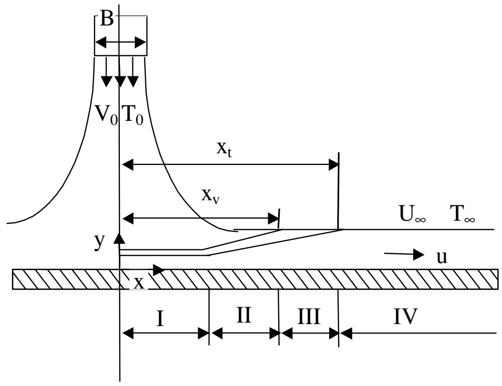
Nomenclature

B	slot nozzle width
C_p	specific heat
H	liquid film thickness
h	heat transfer coefficient
k	thermal conductivity
L	sectional length of slot nozzle
Nu	$=hB/k$, Nusselt number
Pr	$=\mu C_p/k$, Prandtl number
Q	jet flow rate
q	wall heat flux
Re	$=V_0B/v$, Reynolds number
T	local temperature
T_w	local wall temperature
T_0	jet temperature at nozzle exit
T_∞	temperature outside thermal boundary layer
U_∞	velocity outside flow boundary layer
u	lateral velocity component
V_0	jet velocity at nozzle exit
v	vertical velocity component
x	coordinate parallel to impingement plate
x_t	lateral location where thermal boundary layer reaches free surface
x_v	lateral location where viscous boundary layer reaches free surface
y	coordinate normal to impingement plate
Z	nozzle-to-plate spacing

Greek symbols

δ	hydrodynamic boundary layer thickness
Δ	thermal boundary layer thickness
Δ_1	see Eq. (21)
ε	$=\int_0^x q\,\mathrm{d}x/qx$
η	$=y/\delta$
θ	dimensionless temperature
μ	dynamic viscosity
v	kinematic viscosity
ξ	$=\Delta/\delta$
ρ	fluid density
Λ	$=\left(\frac{\delta}{B}\right)^2 Re$
α	$=\frac{Hq}{k(T_w-T_\infty)}$

impingement, but only for the turbulent wall-jet zone. Kendoush [15] studied the heat and mass transfer mechanics of an impinging slot jet by means of boundary-layer theory for laminar flow. The results were restricted just to the stagnation zone. Park [16] and Chiriac [17] reported the numerical solutions of stream and heat transfers in stagnation and wall jet zones for laminar flow, respectively, whereas the solutions to the confined slot jets were available only for a limited range of Reynolds numbers. The objective of the present study is to provide detailed theoretical solutions on laminar flow for free-surface slot jet impinging onto horizontal surfaces under arbitrary-heat-flux conditions. The influence of Prandtl numbers on heat and mass transfers is also considered.



I: 0≤x≤B II: B<x≤x_v III: x_v<x≤x_t IV: x_t<x

Fig. 1. Boundary layers of a free-surface slot jet impinging onto a horizontal plate.

2. Theoretical analyses

The stream of slot jet impingement is shown in Fig. 1, in which the flow field along a heated surface is divided into four regions including a stagnation zone and three wall-jet zones based on the characteristics of flow and heat transfer for $Pr \geqslant 1$, respectively. In this paper predicative formulas are provided for the entire range of lateral flow under arbitrary-heat-flux conditions.

According to boundary layer theory [18], the flow in a liquid film layer satisfies the following conservation equations:

$$\begin{cases} \dfrac{\partial u}{\partial x}+\dfrac{\partial v}{\partial y}=0 \\ u\dfrac{\partial u}{\partial x}+v\dfrac{\partial u}{\partial y}=-\dfrac{1}{\rho}\dfrac{\mathrm{d}P}{\mathrm{d}x}+v\dfrac{\partial^2 u}{\partial y^2} \end{cases} \tag{1–2}$$

with the general conditions:

$$\begin{cases} y=0, \quad u=v=0 \\ y=\delta(y\to\infty), \quad u=U_\infty(x), \quad \dfrac{\partial u}{\partial y}=0 \end{cases} \tag{3–4}$$

$$2L\int_0^H u\,\mathrm{d}y = Q = V_0\cdot(BL) \tag{5}$$

In region (I), stagnation zone, the speed outside the flow boundary layer is

$$U_\infty = V_0 \cdot (x/B) \tag{6}$$

So, the classic four-power polynomial can be used to approximate the velocity profile satisfying Eqs. (3–4):

$$\frac{u}{U_\infty} = f(\eta) = (2\eta - 2\eta^3 + \eta^4) + \frac{\Lambda}{6}\eta(1-\eta)^3 \tag{7}$$

where $\eta = \frac{y}{\delta}$, $\Lambda = \frac{\delta^2}{\nu}U'_\infty = \frac{\delta^2}{B^2}Re$. With the conditions that δ is finite at $x = 0$, δ and can be obtained by integrating Eq. (2):

$$\delta = 2.749BRe^{-0.5}, \quad \Lambda = 7.55 \tag{8}$$

Consequently, we obtain

$$f(\eta) = (2\eta - 2\eta^3 + \eta^4) + 1.260\eta(1-\eta)^3 \tag{9}$$

In region(II), the speed U_∞ remains almost constant, equal to V_0, because the liquid here is unaffected by the viscous stresses. The velocity profile may be expressed by

$$\frac{u}{V_0} = \frac{3}{2}\eta - \frac{1}{2}\eta^3 \tag{10}$$

Substituting Eq. (10) into Eq. (2) yields

$$\frac{d\delta^2}{dx} = \frac{280}{13}\frac{\nu}{V_0} \tag{11}$$

Using the continuous condition of δ between regions (I) and (II) the value of δ in region II can be determined:

$$\delta \cong 2.9517B^{0.281}x^{0.719}Re^{-0.5} \tag{12}$$

Then, the hydrodynamic boundary layer of liquid film flow will be developed until right up to the free surface of stream where lateral distance x_v can be derived from Eqs. (10)–(12):

$$x_v = 0.1627BRe^{0.695} \tag{13}$$

In regions (III) and (IV), as $x > x_v$, the viscous stresses become appreciable right up to the free surface, and the entire flow is of boundary layer type. We are able to use the velocity profile (10), but the free stream speed varies with x. The integral momentum equation should be revised, because in their equation the pressure gradient is zero:

$$\frac{d\delta_2}{dx} + \delta_2\left(2 - \frac{\int_0^H \frac{u}{U_\infty}dy}{\delta_2}\right)\frac{1}{U_\infty}\frac{dU_\infty}{dx} = \frac{\tau_w}{\rho U_\infty^2} \tag{14}$$

From Eq. (5), we have $\frac{5}{4}HU_\infty = BV_0$

Then, we get

$$H = 3.858\frac{x}{Re} + \left(0.8 - \frac{0.6277}{Re^{0.305}}\right)B \tag{15}$$

$$U_\infty = V_0 \bigg/ \left[4.82\frac{x}{Re} + \left(1 - \frac{0.785}{Re^{0.305}}\right)\right] \tag{16}$$

From region (I) to region (III), the four order polynomial are used to approach the temperature profile which satisfies the boundary conditions:

$$\begin{cases} T = T_w, & \dfrac{\partial^2 T}{\partial y^2} = 0, \qquad \text{as } y = 0 \\ T = T_\infty, & \dfrac{\partial T}{\partial y} = \dfrac{\partial^2 T}{\partial y^2} = 0 \quad \text{as } y = \Delta \end{cases} \tag{17–18}$$

The temperature profile is

$$\theta = \frac{T - T_\infty}{T_w - T_\infty} = 1 - 2\frac{y}{\Delta} + 2\left(\frac{y}{\Delta}\right)^3 - \left(\frac{y}{\Delta}\right)^4 \tag{19}$$

The integral energy equation is

$$\frac{\int_0^x q\,dx}{\rho U_\infty C_p} = \Delta_1(T_w - T_\infty) \tag{20}$$

$$\text{where } \Delta_1 = \int_0^\infty \frac{u}{U_\infty}\frac{T - T_\infty}{T_w - T_\infty}dy \tag{21}$$

By the Fourier law, Eq. (19) yields the heat flux at wall

$$q = \frac{2k}{\Delta}(T_w - T_\infty) \tag{22}$$

By solving Eqs. 9 and (19)–(22) simultaneously, a four-order equation for Δ in stagnation zone is obtained

$$\xi^3 - 0.6726\xi^4 = 1.267\frac{\varepsilon}{Pr} \tag{23}$$

where $\xi = \Delta/\delta$, $\delta = 2.749BRe^{-0.5}$ and $\varepsilon = \int_0^x q\,dx/qx$.

The solutions of Eq. (23) with the parameter $Pr\varepsilon^{-1}$ yields, within 5% errors, approximately

$$\xi = \begin{cases} 1.245\varepsilon^{0.47}Pr^{-0.47}; & 1.0 \leqslant Pr\varepsilon^{-1} < 3.0 \\ 1.147\varepsilon^{0.38}Pr^{-0.38}; & 3.0 \leqslant Pr\varepsilon^{-1} < 10.0 \\ 1.082\varepsilon^{1/3}Pr^{-1/3}; & 10.0 \leqslant Pr\varepsilon^{-1} \end{cases} \tag{24}$$

So, $Nu = 0.727\xi^{-1}Re^{0.5}$

$$= \begin{cases} 0.584\varepsilon^{-0.47}Pr^{0.47}Re^{0.5}; & 1.0 \leqslant Pr\varepsilon^{-1} < 3.0 \\ 0.634\varepsilon^{-0.38}Pr^{0.38}Re^{0.5}; & 3.0 \leqslant Pr\varepsilon^{-1} < 10.0 \\ 0.672\varepsilon^{-1/3}Pr^{1/3}Re^{0.5}; & 10.0 \leqslant Pr\varepsilon^{-1} \end{cases} \tag{25}$$

In region (II), Δ can be solved from Eqs. (10), (13), (19), (21) and (22) and obtained:

$$\Delta = 3.894B^{0.76}x^{0.24}\varepsilon Re^{-0.5}Pr^{-1/3} \tag{26}$$

Consequently,

$$Nu = 0.5136\varepsilon^{-1}\left(\frac{B}{x}\right)^{0.24}Re^{0.5}Pr^{1/3} \tag{27}$$

Similar to above calculations, Δ and Nu can be obtained for region (III) as follows:

$$\Delta = 2.924\varepsilon^{1/3}B^{1/3}Pr^{-1/3}Re^{-1/3} \times \left[3.858\frac{x}{Re} + \left(0.8 - \frac{0.6277}{Re^{0.305}}\right)B\right]^{2/3} \tag{28}$$

$$Nu = 0.684\varepsilon^{-1/3}Pr^{1/3}Re^{1/3} \times \left[3.858\frac{x}{ReB} + \left(0.8 - \frac{0.6277}{Re^{0.305}}\right)\right]^{-2/3} \quad (29)$$

From Eqs. (15) and (28), the lateral position x_t can be determined as

$$x_t = \left(\frac{6.48\varepsilon}{Pr} - 0.207Re + 0.163Re^{0.695}\right)B \quad (30)$$

In region (IV), when lateral distance x is larger than x_t, both flow and thermal boundary layers are fully developed through the whole liquid film. In this region, the free surface temperature of liquid film varies with x. Another temperature profile is given by satisfying the boundary condition of heat flux at wall:

$$\theta = 1 - \alpha\frac{y}{H} + \frac{1}{3}\alpha\left(\frac{y}{H}\right)^3 \quad (31)$$

where $\alpha = Hq/[k(T_w - T_\infty)]$.

By similar calculations, we can get

$$Nu = \left[\frac{2\varepsilon}{Pr\,Re} + \frac{272H}{525B}\right]^{-1} \quad (32)$$

where H can be obtained from Eq. (15).

If the heat flux at the wall is uniform, q = constant or $\varepsilon = 1$, the heat transfer coefficients can be simplified from region I to region IV.

3. Comparison of analytical and experimental results

Theoretical expressions of heat transfer coefficients for free-surface slot jets were obtained by undertaking the mathematical analyses shown above. Comparisons of heat transfer coefficients in both the stagnation zone and the wall-jet zone were made between the present study and the analytical and experimental results in published literature.

Heat transfer coefficients at the stagnation line for slot jet impingement are shown in Fig. 2, in which both equations from this study and expressions from other researches are plotted. McMurray et al. [19], Miyasaka et al. [20], Zumbrunnen et al. [21] and Vader et al. [22] presented either analytical or empirical correlations of heat transfer for free-surface slot jet impingement. In their experiments water was mostly used as the working fluid together with nozzles of different slot widths. Good agreements between present theoretical results and the correlations from other studies were observed even for the high Reynolds numbers ($Re > 10000$). A deviation below 25% between the present equations and other empirical correlations exists except for those by Zumbrunnen with $B = 10.2$ mm due to different turbulence level of jet flow. Another plotted line based on the theoretical equation by Kendoush [15], also shown in the figure, shows somewhat a higher Nusselt numbers than that of present theoretical results within a deviation of 27%. The relative deviations can be explained by the influence of the turbulent intensity of the free jet in those experiments. The level of turbulent flow at the exit of the nozzle is distinctly different for varied Prandtl numbers, Reynolds numbers and geometry of nozzles.

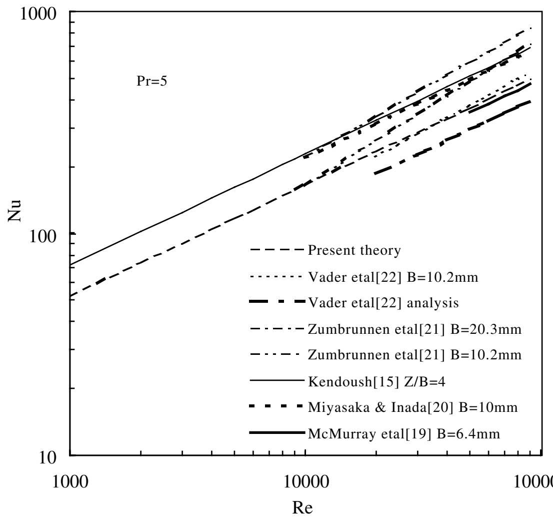


Fig. 2. Comparisons of stagnation heat transfer for free-surface slot jet impingement.

Figs. 3 and 4 exhibit comparisons of local heat transfer coefficients along heated surfaces between present results and the experimental data by Vader et al. [22], Zumbrunnen et al. [21]. The experimental results of Vader et al. [22] and Zumbrunnen et al. [21] with water as the working fluid are shown in Figs. 3 and 4, in which

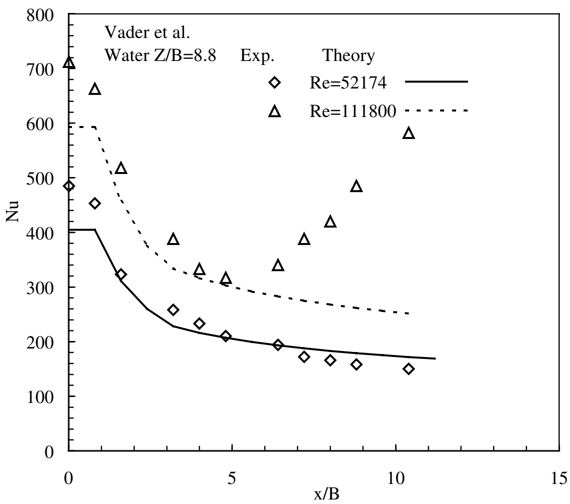


Fig. 3. Comparison of theoretical results with experimental results by Vader et al. [22] with water.

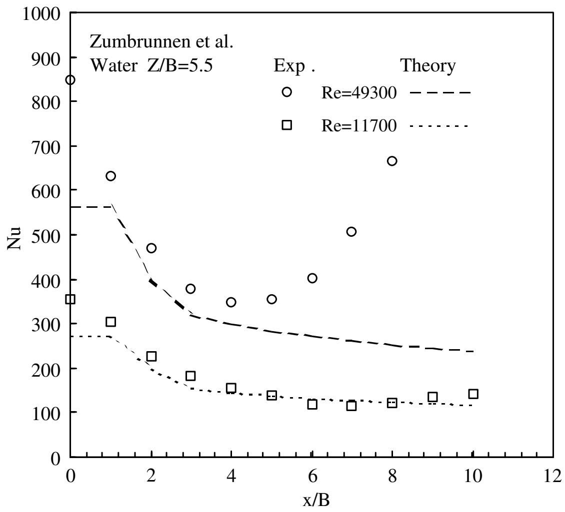


Fig. 4. Comparison of theoretical results with experimental results by Zumbrunnen et al. [21] with water.

generally good agreements between experimental results and the present correlations are observed before the transition from laminar to turbulent flow. As illustrated in the figures, the humps in heat transfer distribution curves are clearly observable from the experimental results, and are due to the transition at flow boundary layer from laminar flow to turbulence. The local heat transfer may be well regulated by the present theory, focused on the laminar flow of boundary layer, even for initially turbulent impinging jet with high Reynolds numbers ($Re > 10\,000$).

4. Conclusion

A theoretical analysis has been conducted to study heat transfer characteristics of free-surface liquid slot jet impingement. Equations for local heat transfer coefficients are obtained along heated surfaces with slot jets impingement based on the theory of laminar boundary layer. Also, the formula is compared with experimental results for both different working fluids and different nozzle sizes. Present theory shows good agreements with experimental results of for slot jet impingement. Further research should be developed to correlate heat and mass transfers of slot jet impingement with high turbulence.

Acknowledgements

The present research is supported by National Key Fundamental Research Plan (973) Project of China (no. 2000026304) and National Natural Science Foundation of China.

References

[1] H. Martin, Heat and mass transfer between impinging gas jets and solid surfaces, in: j.p. hartnett, t.f. irvine (Eds.), Adv. Heat Transfer, Vol. 13, Academic Press, 1977, pp. 1–60.

[2] S.J. Dovns, E.H. James, Jet Impingement Heat Transfer–A literature Survey, ASME paper 87-HT-35, 1987.

[3] M. Kiryu, Development of oil-cooled 750 cc motorcycle engine, Automobile Technol. (in Japanese) 40 (9) (1986) 1154–1158.

[4] C.F. Ma, A.E. Bergles, Convective heat transfer on a small vertical heated surface in an impingement circular liquid jet, in: B.X. Wang (Ed.), Heat Transfer Science and Technology 1988, Hemisphere, 1990, pp. 193–200.

[5] C.F. Ma, Fundamental research on extremely small size liquid jet impingement heat transfer, in: Proceeding of 3rd International Thermal Energy Congress, Kitakyushu Japan, July 28–August 1, 1997, pp. 195–202.

[6] L.M. Jiji, Z. Dagan, Experiment investigation of single phase multi-jet impingement cooling of an array of microelectronic heat sources, in: Win Aung (Ed.), Cooling Technology for Electronic Equipment, Hemisphere publishing Corp., 1988, pp. 332–352.

[7] C.F. Ma, A.E. Bergles, Boiling jet impingement cooling of simulated microelectronic chips, ASME Publ. HTD 28 (1983) 5–12.

[8] F.C. Kohing, Waterwall: Water cooling system, Iron-Steel Eng. 62 (June) (1985) 30–36.

[9] C.F. Ma, J. Yu, D.H. Lei, Y.P. Gan, F.K. Tsou, H. Auracher, Transient jet impingement boiling heat transfer on hot surfaces, in: X.J. chen et al. (Eds.), Multiphase Flow and Heat Transfer-second International Symposium, vol. 1, Hemisphere, 1990, pp. 349–357.

[10] X. Liu, V.J.H. Lienhard, J.S. Lombara, Convective heat transfer by impingement of circular liquid jets, J. Heat Transfer 113 (1991) 571–582.

[11] C.F. Ma, Y.H. Zhao, T. Masuoka, T. Gomi, Analytical study on impingement heat transfer with single-phase free-surface circular liquid jets, J. Thermal Sci. 5 (4) (1996) 271–277.

[12] Y. Zhuang, C.F. Ma, M. Qin, Experimental study on local heat transfer with liquid impingement flow in two-dimensional micro-channels, Int. J. Heat Mass Transfer 40 (17) (1997) 4055–4059.

[13] Z.H. Lin, Y.J. Chou, Y.H. Hung, Heat transfer behaviors of a confined slot jet impingement, Int. J. Heat Mass Transfer 40 (5) (1997) 1095–1107.

[14] W.H. Schwarz, W.P. Cosart, The two-dimensional turbulent wall-jet, J. Fluid Mech. 10 (1961) 481–495.

[15] A.A. Kendoush, Theory of stagnation region heat and mass transfer to fluid jets impinging normally on solid surfaces, Chem. Eng. Process. 37 (1998) 223–228.

[16] V.A. Chiriac, A. Ortega, A numerical study of the unsteady flow and heat transfer in a transitional confined slot jet impinging on an isothermal surface, Int. J. Heat Mass Transfer 45 (2002) 1237–1248.

[17] T.H. Park, H.G. Choi, J.Y. Yoo, S.J. Kim, Streamline upwind numerical simulation of two-dimensional confined impinging slot jets, Int. J. Heat Mass Transfer 46 (2003) 251–262.

[18] H.Z. Liu, H.J. Shu, Theory of Boundary Layer, People's Jiaotong Press of China, 1991.
[19] D.C. McMurray, P.S. Myers, O.A. Uychara, Influence of impinging jet variables on local heat transfer coefficients along a flat surface with constant heat flux, in: Proc. 3rd Int. Heat Transfer Conf., New York, AIChE J 2, 1966, pp. 292–299.
[20] Y. Miyasaka, S. Inada, The effect of pure forced convection on the boiling heat transfer between a two-dimensional subcooled water jet and a heated surface, J. Chem. Eng. Japan 15 (1980) 22–28.
[21] D.A. Zumbrunnen, F.P. Incropera, R. Viskanta, Convective heat transfer distribution on a plate cooled by planar water jets, J. Heat Transfer 111 (1989) 889–896.
[22] D.T. Vader, F.P. Incropera, R. Viskanta, Local convective heat transfer from a heated surface to an impinging planar jet of water, Int. J. Heat Mass Transfer 34 (1991) 611–623.